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A Mathematical Electron

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A particle (m) is represented by a Ricci-flat Schwarzschild-based *analytic enerel ("energy + i × electric charge") geometry X* with fundamental form: $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta = (1-2m/r)dt^2 - (1-2m/r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$, where $x^\gamma = (t, r, \theta, \varphi)$, $m = \mathbf{m} + i\mathbf{e}$ is *enerel*, $\mathbf{m}$ is rest energy (or mass), and $\mathbf{e}$ is electric charge. A unitary vector u_α and a scalar ϕ are defined in X by means of the postulated constitutive equation: $\phi u_{\alpha\beta}u^\beta + \phi_\alpha = 0$. The normalization condition is postulated as:

$$m = \kappa \int_0^\pi \int_0^\pi \int_{-a}^{a} \phi \rho g^{1/2} u^0 dr\, d\theta\, d\varphi$$

where $\rho = (R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta})^{1/2}$ is an enerel density function, $\kappa = m^2/\hbar c$ is a normalization constant and also a fine-structure constant, $g = \det g_{\alpha\beta} = -r^4\sin^2\theta$, and $R_{\alpha\beta\gamma\delta}$ is the Riemann curvature tensor formed with complex $g_{\alpha\beta}$. These equations yield the charge $\mathbf{e} = 0.0855115\ldots$ of a purely mathematical *eltron* $(m) = (i\mathbf{e})$ of radius a.

1. INTRODUCTION

A specific *analytic enerel (energy + i × electric charge) geometry X* is proposed to represent a mathematical electron; X has a fundamental form $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$ and is defined in this paper by means of postulated partial differential equations and other conditions. The geometry X is based on a Schwarzschild geometry, i.e., X is a Ricci-flat geometry with $R_{\alpha\beta} = 0$, where $R_{\alpha\beta} = R^\mu_{\alpha\beta\mu}$, $R_{\alpha\beta}$ denotes the Ricci curvature tensor, and $R^\alpha_{\beta\gamma\delta}$ denotes the Riemann curvature tensor formed with the metric tensor $g_{\alpha\beta}$ of X; $\alpha, \beta, \gamma, \ldots = 0, \ldots, n-1$.

The postulates of an analytic enerel geometry $X' = (g_{\alpha\beta}, u_\alpha, \phi; \rho)$ set forth in this paper are based on those of an analytic enerel (Gleyzal, 1976) geometry $Z=(z_{\alpha\beta}, u_\alpha, \phi; \rho)$ of n independent coordinate variables $z^\gamma = x^\gamma + iy^\gamma$ and enerel parameters $m_K = \mathbf{m}_K + i\mathbf{e}_K$; $K=0,\ldots,N-1$; $\gamma=0,\ldots,n-1$; $n=1,2,3,\ldots$; $N=0,1,2,\ldots$. In this paper, $n=4$, $K=0$, $N=1$.

Absolute units of measurement are used, where $\gamma=c=\hbar=1$, γ is the gravitational constant, c is the velocity of light, $\hbar=h/2\pi$, and h is Planck's constant. We note $m^2\gamma/\hbar c$ is dimensionless if m has the dimension of mass.

By postulate, a geometry X' possesses in addition to the metric tensor $g_{\alpha\beta}$ a velocity or "streamline" vector $u^\alpha = dz^\alpha/ds$, a scalar energy potential function ϕ, and a scalar energy density function ρ. The tensor $g_{\alpha\beta}$, vector u_α, and scalars ϕ and ρ may be either real valued or complex valued in a geometry X', but the coordinates x^γ are assumed to be real valued in X'.

In terms of a spherically symmetric geometry X, a purely mathematical derivation $\mathbf{e}=0.0855115\ldots$ of the electric charge of an *eltron* ($i\mathbf{e}$) is proposed. This number can be compared with the experimentally determined charge $e_+ = 0.08542$ of an electron.

2. DERIVATION OF AN ELTRON ($i\mathbf{e}$)

We begin with the well-known Schwarzschild metric: $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$, where

$$ds^2 = \left(1-2\frac{m}{r}\right)dt^2 - \left(1-2\frac{m}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (2.1)$$

and $x^\gamma = (t, r, \theta, \varphi)$. This metric is associated with a particle (m) of energy (or mass) m, where m is an arbitrary integration constant and may be real or complex.

A *unitary velocity vector* field $u^\alpha = dx^\alpha/ds$, where $u_\alpha = g_{\alpha\beta}u^\beta$, $u_\alpha u^\alpha = 1$, and $u^k = 0$ is assumed to exist in the geometry X. There results

$$u^0 = \left(1-2\frac{m}{r}\right)^{-1/2}, \qquad u_0 = \left(1-2\frac{m}{r}\right)^{1/2}, \qquad u_k = 0 \qquad (2.2)$$

in the geometry X; $k=1, 2, 3$.

A scalar *energy potential function* ϕ is postulated to exist in any analytic enerel geometry X'. In the geometry X:

$$\phi = \left(1-2\frac{m}{r}\right)^{1/2} \qquad (2.3)$$

Given the metric tensor $g_{\alpha\beta}$ of X' subject to the constitutive equation $R_{\alpha\beta}=0$, the constitutive equation which defines u_α and ϕ in a general geometry X' is postulated to be

$$\phi u_{\alpha\beta}u^\beta+\phi_\alpha=0 \tag{2.4}$$

It is readily verified that ϕ and u_α as expressed by equations (2.2) and (2.3) for the particular geometry X constitute a solution of equation (2.4) for general enerel geometries X'; for

$$u_{\alpha\beta}u^\beta=(u_{\alpha\beta}-u_{\beta\alpha})u^\beta=(u_{\alpha 0}-u_{0\alpha})u^0=(u_{\alpha,0}-u_{0,\alpha})u^0$$

$$=-u_{0,\alpha}u^0=-u_{0,\alpha}/u_0=-\phi_{,\alpha}/\phi$$

where $f_{,\alpha}$ signifies $\partial f/\partial x^\alpha$, and λ_α signifies the covariant derivative of a scalar, vector, or tensor λ with respect to the coordinate x^α. Therefore, $\ln\phi=\ln u_0$; consequently $\phi=u_0=[1-2(m/r)]^{1/2}$ is a solution of equation (2.4) in the particular geometry X. In summary: equations (2.1), (2.2), and (2.3) express an exact solution X of the constitutive equations of an analytic enerel geometry X'.

In conventional theory it is usually assumed that the energy density ρ vanishes if $R_{\alpha\beta}=0$ except on a singularity. Let us depart from this view as follows. Given the metric tensor $g_{\alpha\beta}$ of X', we write

$$\rho=\left(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}\right)^{1/2} \tag{2.5}$$

Thus ρ is categorically expressed in terms of $g_{\alpha\beta}$ and the first and second derivatives of $g_{\alpha\beta}$ with respect to the coordinate variables x^γ. This form of definition of energy density (non-complex-valued) has been considered by Eddington (1937, p. 141) in his analysis of the problem of choosing the field equations of gravitational relativity theory. In the case where $g_{\alpha\beta}$ is the Schwarzschild tensor,

$$\rho=(48)^{1/2}m/r^3 \tag{2.6}$$

By postulate, the total mass M due to density ρ and potential energy ϕ in a sphere of radius a of the geometry X is given by the invariant integral:

$$M=\kappa\int_0^\pi\int_0^\pi\int_{-a}^{a}\rho\phi u^0 g^{1/2}dr\,d\theta\,d\varphi \tag{2.7}$$

where κ is a universal constant, and $g=\det g_{\alpha\beta}$. It is of interest that in corresponding integrals of conventional theory there is introduced, *ad hoc*, a factor $(-g)^{1/2}=ig^{1/2}$ in order to "eliminate" i. By calculation in X: $u^0\phi=1$, and $g=-r^4\sin^2\theta$. Therefore

$$M=\kappa\int_0^\pi\int_0^\pi\int_{-a}^a i(48)^{1/2}\frac{m}{r}\sin\theta\,dr\,d\theta\,d\varphi \tag{2.8}$$

expresses the mass M in a sphere of radius a due to a particle of mass m which generates the field $g_{\alpha\beta}$.

Let the symbol m, where $m=\mathbf{m}+i\mathbf{e}$, $\mathbf{m}$ is mass or energy, and $\mathbf{e}$ is electrical charge, now denote *enerel* in a new unified field theory; cf. Moffat (1958) and Gleyzal (1976). The physical meaning of the various quantities may then change and require re-identification; for convenience of exposition the same words or symbols may be used but with a different meaning.

Evaluation of the integral (2.8) yields

$$\begin{aligned} M&=\kappa i^2(48)^{1/2}\cdot 2\pi^2 m \\ &=-\kappa(48)^{1/2}\cdot 2\pi^2 m \end{aligned} \tag{2.9}$$

Let us assume $M=m$. In this case the quantity m cancels out, and there results the normalization condition:

$$\begin{aligned} 1&=-\kappa(48)^{1/2}\cdot 2\pi^2 \\ &=-\kappa 136\cdot 75723\ldots \end{aligned} \tag{2.10}$$

Therefore

$$\kappa=-1/136.75723\ldots \tag{2.11}$$

This number, except for sign, is close in value to the fine-structure constant 1/137.037 of quantum mechanical theory. Either this agreement of the two numbers is pure chance and the proposed theory of enerel mathematical physics is simply another unified field theory, or, the foregoing derivation of the fine-structure constant as a normalization condition is potentially a significant new turning point in mathematical physics.

Accordingly, let us write

$$m^2=\kappa$$

Then $m=i\cdot(.0855115\ldots)$ is the "energy" of an *eltron* $(m)=(i\mathbf{e})$.

This result suggests that the particle described by equations (2.1), (2.2), (2.3), and (2.6) represents an eltron, a massless, pure electric charge.

Note. A general theory of analytic enerel geometry $Z = (Z^n; z_{\alpha\beta}, u_\alpha, \phi; \rho; \dot{\psi}^\alpha, \gamma^{\alpha\beta}_{\gamma}; m_K)$ will be offered for publication. This theory includes an "enerel generalization" of the Dirac–Bargmann theory (Brill and Wheeler, 1957) of wave mechanics.

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